THE VIRIAL MASS OF THE NUCLEUS OF M32

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Abstract. By means of the virial theorem we derive the dependence of the mass of an oblate spheroid in solid body rotation from the velocity dispersion and the space light density. The latter is obtained from a calibrated and seeing deconvolved brightness profile as numerical and stable solution of the Abel integral equation. The application of the nucleus of M32 gives a central density of $2.1 \times 10^{-5}\, M_\odot\, \mathrm{pc}^{-3}$, a nuclear mass of $4.3 \times 10^{-7}\, M_\odot$ and a mass-to-light ratio of 4.6 in V-band.

1. Introduction

It is known that, under good seeing conditions, the nuclei of M31 and M32 show a central spike, some arc sec in diameter, defined by a sudden change in the slope of the inner brightness profile. Owing to the solid body rotation in their interior (Lallemand *et al.*, 1960; Walker, 1962, 1974) these nuclei are believed to be dynamically separate components of the galaxies.

Far from the local group similar peaks, if they exist in external galaxies, cannot be seen by ground based observations since their apparent angular diameters become much smaller than the dispersion parameter of the gaussian seeing (see, e.g., Schweizer, 1979).

The physical parameters of the nucleus of M31 were derived by Light *et al.* (1974) fitting the data of the Stratoscope II balloon-borne telescope by the brightness distribution of an isothermal gas sphere convolved with the relative instrumental point-spread function, and using available values of the line-of-sight nuclear velocity dispersion.

The purpose of this paper is the determination of the same parameters in M32 by means of the virial theorem using only ground based measurements numerically processed without assumption of a model of light distribution.

2. The Virial Mass Estimate

We consider an oblate spheroid, with matter density $\rho(r)$ and light density $\nu(r)$ along the semi-major axis, in solid body rotation with angular velocity around the minor axis. We will further assume that the squared velocity dispersion $\bar{v}^2$, the mass-to-light ratio and the axial ratio k (or the ellipticity e) constant within the spheroid of equatorial radius R. Then the virial theorem can be expressed by

$$M(R)\bar{v}^2 + I(R)\omega^2 = [G \sin^{-1} e/2e] \int_0^R \frac{M(r)\,\mathrm{d}M(r)}{r}\,, \tag{1}$$

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where the moment of inertia (see, e.g., Routh, 1877) is

$$I(R) = \frac{8}{3}\pi k \int_0^R r^4 \rho(r)\,\mathrm{d}r\,. \tag{2}$$

Let $L_\nu(r)$ be the integrated luminosity of the spheroid of equatorial radius r – i.e.,

$$L_\nu(r) = 4\pi k \int_0^r r^2 \nu(r)\,\mathrm{d}r\,. \tag{3}$$

From the above assumptions we can write

$$\rho(r) = \frac{M(R)}{L(R)}\,\nu(r) \quad \text{and} \quad M(r) = \frac{M(R)}{L(R)}\,L_\nu(r)\,. \tag{4}$$

Making the substitutions in (1) and simplifying, we derive the expression for the mass inside R of the form

$$M(R) = \frac{L_\nu^2(R)\bar{v}^2 + (\frac{2}{3})L_\nu(R)\omega^2 \int_0^R 4\pi k r^4 \nu(r)\,\mathrm{d}r}{(2\pi G k \sin^{-1} e/e) \int_0^R L_\nu(r)\nu(r) r\,\mathrm{d}r} \tag{5}$$

3. Photometric Structure and Mass of the M32 Nucleus

To derive from Equation 5 the mass and mass-to-light ratio of the M32 nucleus, ellipticity, rotation and velocity dispersion are taken from current literature values. As far as the required space light density $\nu(r)$ is concerned, it is obtained solving the Abel integral equation

$$\sigma(x) = 2\int_x^\infty \frac{r\nu(r)\,\mathrm{d}r}{(r^2 - x^2)^{1/2}} \tag{6}$$

by the Tikhonov method (see, e.g., Tikhonov and Arsenine, 1976); in which $\sigma(x)$ is the photoelectric calibrated brightness profile along the semi-major axis previously obtained (Bendinelli *et al.*, 1978) and deconvolved from seeing (Bendinelli *et al.*, 1982).

The numerical values in this paper were specified assuming the basic quantities reported in Table I.

The inner photometric structure of M32 is reported in Table II. The first column gives the mean equatorial radius of the elliptical anuli or ellipsoidal shells by which the Galaxy continuous structure was discretized to derive the distribution of the surface bright-

TABLE I

Assumed constants for M32 and Sun

Distance modulus	24.2
V absolute magnitude	– 16.34
$B - V$	1.0
V absolute magnitude of the Sun	4.83
$B - V$ of the Sun	0.57

ness σ (column 2) and the space light density ν (column 3). The behaviour of column 2 of this table was previously published in Bendinelli *et al.* (1982). The last two columns give the luminosities L_σ and L_ν, obtained integrating the brightness out to the external radius of the respective anulus or shell.

The comparison between the photometric nuclear parameters of M32 (in parenthesis the values uncorrected from seeing) and M31 (from Light *et al.*, 1974) is presented in Table III.

The physical parameters of the M32 nucleus, derived from Equation 5, depend on the square of the velocity dispersion as well as those in the model of M31. Hence, for a suitable comparison of the nuclei we use the pair 96 and 192 km s^{-1} of the central line-of-sight velocity dispersion σ^2 (with $\bar{v}^2 = 3\sigma^2$), taken from the survey of Torny and Davis (1981). The rotation of M32 (Walker, 1962) has not, in practice, sensible effects on the estimate of the physical parameters. The calculated ratio of rotational to random energy found in the nucleus of M32 is about $0.05 v_{\mathrm{rot}}/v_{\mathrm{ran}}$, in agreement with the results of Bertola and Capaccioli (1975) and Peterson (1978) showing that elliptical galaxies are mainly pressure supported.

A rough estimate of the total mass of M32 can be made assuming its proportionality, via the nuclear mass-to-light ratio, to the total extrapolated luminosity. The discrepancy of this mass ($12.8 \times 10^8 M_\odot$) with the result of $18.9 \times 10^8 M_\odot$ ($M/L_\mathrm{B} = 10$) of Torny and Davis (1981) obtained using the method developed by Poveda (1958), in our opinion seems to confirm the hypothesis of the dynamically separate nature of the nucleus. In both these estimates the same central velocity dispersion, the same total luminosity and the assumption of isotropy of the mass-to-light ratio have been used.

4. Concluding Remarks

It is known that the velocity dispersion and the rotation measurements of elliptical galaxies far from the centre are not so detailed and sound to permit a reliable estimate of the mass and the mass-to-light ratio via the physically well founded hydrodynamical model. To avoid this lack of measurements, two approximate methods, one due to Poveda (1958); the other to King and Minkowski (1972), are used in practice. Both these methods assume a priori the distribution of the light (and hence of the mass), according to the empirical laws of de Vaucouleurs (1953) and King (1962), respectively.

The present method, in which the light distribution is not constrained a priori, should give a better approximation for the mass estimate.

TABLE II

The V inner photometric structure of M32

r arc sec	V mag arc sec^{-2}	$\nu(r)$ $L_\odot$ pc^{-3}	$L_\sigma(r)$ $L_\odot$	$L_v(r)$ $L_\odot$
0.06	13.12	0.4741E+05	0.1330E+06	0.8660E+04
0.17	13.20	0.4256E+05	0.5035E+06	0.7862E+05
0.29	13.28	0.3774E+05	0.1074E+07	0.2509E+06
0.40	13.38	0.3306E+05	0.1807E+07	0.5468E+06
0.52	13.48	0.2866E+05	0.2665E+07	0.9708E+06
0.64	13.59	0.2465E+05	0.3611E+07	0.1516E+07
0.75	13.71	0.2106E+05	0.4613E+07	0.2166E+07
0.87	13.84	0.1787E+05	0.5640E+07	0.2900E+07
0.98	13.98	0.1501E+05	0.6664E+07	0.3692E+07
1.10	14.12	0.1244E+05	0.7664E+07	0.4513E+07
1.21	14.27	0.1013E+05	0.8627E+07	0.5329E+07
1.33	14.43	0.8077E+04	0.9543E+07	0.6109E+07
1.44	14.58	0.6285E+04	0.1041E+08	0.6827E+07
1.56	14.72	0.4768E+04	0.1123E+08	0.7462E+07
1.67	14.84	0.3525E+04	0.1202E+08	0.8003E+07
1.79	14.95	0.2553E+04	0.1278E+08	0.8451E+07
1.91	15.03	0.1834E+04	0.1354E+08	0.8816E+07
2.02	15.08	0.1339E+04	0.1430E+08	0.9116E+07
2.14	15.11	0.1036E+04	0.1509E+08	0.9375E+07
2.25	15.12	0.8898E+03	0.1591E+08	0.9622E+07
2.37	15.12	0.8622E+03	0.1677E+08	0.9887E+07
2.48	15.11	0.9189E+03	0.1768E+08	0.1020E+08
2.60	15.10	0.1028E+04	0.1864E+08	0.1058E+08
2.71	15.09	0.1163E+04	0.1965E+08	0.1105E+08
2.83	15.09	0.1302E+04	0.2071E+08	0.1162E+08
2.95	15.09	0.1427E+04	0.2181E+08	0.1230E+08
3.06	15.10	0.1528E+04	0.2294E+08	0.1308E+08
3.18	15.13	0.1597E+04	0.2409E+08	0.1396E+08
3.29	15.15	0.1631E+04	0.2525E+08	0.1493E+08
3.41	15.19	0.1631E+04	0.2641E+08	0.1597E+08
3.52	15.24	0.1601E+04	0.2756E+08	0.1706E+08
3.64	15.29	0.1545E+04	0.2870E+08	0.1818E+08
3.75	15.34	0.1468E+04	0.2981E+08	0.1931E+08
3.87	15.40	0.1378E+04	0.3090E+08	0.2044E+08
3.98	15.46	0.1279E+04	0.3196E+08	0.2155E+08
4.10	15.52	0.1178E+04	0.3299E+08	0.2264E+08
4.22	15.59	0.1077E+04	0.3398E+08	0.2368E+08
4.33	15.65	0.9817E+03	0.3495E+08	0.2469E+08
4.45	15.71	0.8944E+03	0.3590E+08	0.2566E+08
4.56	15.76	0.8165E+03	0.3682E+08	0.2659E+08
4.68	15.81	0.7485E+03	0.3772E+08	0.2749E+08
4.79	15.86	0.6908E+03	0.3860E+08	0.2836E+08
4.91	15.91	0.6420E+03	0.3946E+08	0.2921E+08
5.02	15.95	0.6025E+03	0.4031E+08	0.3004E+08
5.14	15.99	0.5703E+03	0.4115E+08	0.3086E+08
5.26	16.03	0.5444E+03	0.4198E+08	0.3169E+08
5.37	16.07	0.5225E+03	0.4279E+08	0.3251E+08
5.49	16.11	0.5039E+03	0.4360E+08	0.3334E+08
5.60	16.14	0.4876E+03	0.4439E+08	0.3418E+08
5.72	16.18	0.4722E+03	0.4517E+08	0.3503E+08
5.83	16.22	0.4569E+03	0.4594E+08	0.3588E+08
5.95	16.27	0.4412E+03	0.4669E+08	0.3673E+08
6.06	16.31	0.4248E+03	0.4743E+08	0.3759E+08

TABLE III

Photometric parameters of the nuclei

	M32	M32
r (arc sec)	2.1	2.0
b/a	0.70	0.63
$\sigma(0)$ (V mag. arc sec^{-2})	13.1 (13.7)	12.7
$\nu(0)$ ($10^4\ L_\odot$ pc^{-3})	4.7 (2.2)	14.3
m (V mag.)	12.0 (12.0)	12.6
L ($10^6\ L_\odot$)	9.4 (9.4)	5.3

TABLE IV

Physical parameters of the nuclei

	ρ	M	M/L_v
	$10^5\ M_\odot$ pc^{-3}	$10^7\ M_\odot$	$M_\odot/L_\odot$
M32	2.1	4.3	4.6
M31	30.9	11.6	21.7

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